Journal of Structural Engineering & Applied Mechanics (2024) 7(1):58-68
DOI 10.31462/jseam.2024.01058068

RESEARCH ARTICLE

Mathematical simulation of transverse cut in the fiber of a composite reinforced with unidirectional fibers with linear cohesion cracks

Rafail Mehtiyev[1*]

[1] Azerbaijan State Petroleum and Industry University, Scientific Research Institute "Geotechnical Problems of Oil, Gas and Chemistry", Baku, Azerbaijan

Article History

Received 4 October 2023
Accepted 20 March 2024

Keywords

Composite material
Transverse slip
Singular integral equation
Cohesive crack
Reinforcement fibers

Abstract

In the reviewed paper, the case of a transverse displacement plane with two pairs of biperiodic (corresponding to the X and Y axes) cohesive cracks of unequal size, weakened by biperiodic circular holes, is considered. The circular holes are filled with reinforcing fibers and the surface is covered with a thin homogeneous non-metallic material of the same thickness. In this case, boundary issues between filler and coating, coating and matrix, and boundary issues along cohesion cracks are determined. For both cases, the solution to the problem is sought in the form of an analytical function with complex variables. According to the boundary conditions of the case, a system of unequal algebraic equations is found along the holes, and singular integral equations are constructed along the cohesion cracks. Currently, the singular integral equations are brought to the system of finite linear algebraic equations with the help of mathematical transformations. Both systems are solved together using the Gaussian method and the crack growth is determined using the stress intensity factor formulas at the crack tips. During the solution of the problem, the stress intensity coefficients at the end are found by the variation of the length of the cracks based on the radius of the circular holes.

1. Introduction

In the problems of plane mechanics, the mechanics of collapse are of great importance, and from this point of view, the investigation of the collapse of structures is of particular importance. Thus, there are certain defects in the operation of the military construction. This reduces the service life of the structure. In this regard, the following issue is of particular importance.

Currently, technical means in the form of perforated elements are used in many branches of modern technology. In this context, it is of great importance to develop methods for calculating the strength of perforated elements of cracked machines and structures [13]. The study of these issues is important in connection with the development of energy, the chemical industry, and other branches of technology, as well as the widespread use of materials with a double periodic structure (composites).

* Corresponding author (rafail60mehtiyev@mail.ru)
eISSN 2630-5763

A fairly complete picture of the characteristic stress distribution in the microstructure of linearly reinforced materials can be obtained by examining the shear stress distribution in a plane perpendicular to the fiber orientation. The solution to this problem opens new possibilities for predicting the mechanical properties of composite materials based on the initial properties given for the constituent components and microstructure type.

The solution to contact problems is extremely important as we encounter many situations involving such problems in our daily lives [15]. One of the most important parameters effective in solving contact problems is the materials of the parts in contact. While it is relatively easy to solve the contact mechanics of the systems created with traditional materials with a homogeneous microstructure and mechanical distribution, it may be more difficult to solve the contact problem of new-generation materials that do not show a homogeneous distribution [12].

Contact mechanics analysis is crucial because such problems often arise in engineering practice. When examining contact mechanics, the material property of the contacting components is a crucially significant aspect [16]. It is more complex to solve the contact mechanics of systems that are composed of materials that do not have a homogenous structure compared to materials that have homogeneous qualities throughout. While many studies on contact problems with homogeneous materials exist, those involving non-homogeneous materials are scarce in the literature. As material technology improves fast, there will be a greater need to solve such problems.

These types of issues are reflected in modern spaceships, airplanes, and equipment developed by the Ministry of Defense Industry. The discussed issue can be considered relevant for modern times [11].

2. Field equations and formulation

A model of fracture of composite materials with a doubly periodic structure is proposed, based on consideration of the zone of the fracture process near the crack tip. It is accepted that the zone of the destruction process is a layer of finite length containing material with partially broken bonds between its individual structural elements (end zone), considered as part of the crack. The presence of bonds between the crack faces in the end zone is modeled by applying adhesion forces caused by the presence of bonds to the crack surfaces. The analysis of the limit equilibrium of a cohesive crack under longitudinal shear is performed based on the criterion of the limiting shear of material bonds [7].

Let there be an isotropic medium weakened by a periodic system of circular holes having radius λ $(\lambda < 1)$ and centers at points

$$P_{mn} = m\omega_1 + n\omega_2, (m, n = 0, \pm 1, \pm 2, \ldots,)$$
$$\omega_1 = 2, \qquad \omega_2 = \omega_1 h e^{i\alpha}, h > 0, \quad \operatorname{Im}\omega_2 > 0 \tag{1}$$

The circular holes of the plane (binder) are filled with washers (linearly reinforced with fibers) made of a homogeneous elastic material. The plane is weakened by two systems of periodic linear cracks with end zones connected between the faces. End zone crack models have been proposed for brittle materials [1], and end zone models in the case of plastic flow at constant stress have been considered in [2,3].

The rows (edges) of shear cracks outside the end zones are free from external loads and $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau_{xy}^{\infty}$, (shear at infinity) stresses act on the composite body (Fig. 1).

Note that an increase in external load following the cracks will cause pre-fracture zones to form. A crack model is used with connections between edges at the end regions of the pre-fracture. The tip regions of cracks are modeled by areas with weakened interparticle bonds in the material. If the length of the crack tip is not small relative to the length of the crack, then methods for evaluating the fracture resistance of a material by considering a small-tip crack are not applicable. In these cases, it is necessary to simulate the stress state in the tip region of the crack using the deformation characteristics of the bonds and a two-parameter failure criterion that accounts for both the evolution of the crack tip and the change. in the size of the tip region of the crack during its growth. If the deformation and fracture processes in the tip zones of cracks involve several physical mechanisms, such as in composite materials, in such cases it is effective to use the tip zone model with a singularity at the crack tip [8,9].

For homogeneous materials, such a normal fracture crack model in articles is considered, and development is given for cracks with an end zone at the interface of materials with different properties.

Modeling of end zones consists of treating them as part of the cracks and explicitly applying cohesive forces to the surface of the cracks in the end zones, which prevent them from sliding. It is accepted that the dimensions of the tip areas of the cracks are proportional to the length of the cracks. The interaction of the edges of the end zones is modeled by adding a pre-fracture zone between the edges of the bonds with a given deformation diagram. The physical nature of such bonds and the size of the pre-fracture zones depend on the type of material [14]. Under the influence of an external load on a composite body, tangential forces $q_x(x)$ and $q_y(y)$ will occur in the ties connecting the edges of the end zones of the cracks. These stresses are not known in advance and must be determined and reduced to the structure in two functions and $\Psi(z)$ in each of the areas occupied by the medium according to the conditions given at the boundaries of the elastic medium. $\Phi(z)$ The boundary conditions in the problem considered are as follows:

$$(\sigma_r - i\tau_{r\theta})_{b|\Omega_m} = (\sigma_r - i\tau_{r\theta})_{s|\Omega_m} \quad \text{parallel to the } x\text{-axis}$$
$$(u + iv)_{b|\Omega_m} = (u + iv)_{s|\Omega_m} \quad \text{parallel to the } y\text{-axis} \tag{2}$$

on the edge of cracks with end zones

$$\left(\sigma_y - i\tau_{xy}\right)_s = f_x(x) \quad \text{parallel to the } x\text{-axis}$$
$$\left(\sigma_x - i\tau_{xy}\right)_s = f_y(y) \quad \text{parallel to the } y\text{-axis} \tag{3}$$

where Ω_m m fiber-matrix interface in the cell number, the quantities related to coating, flake, and plane are shown with b and s indices, respectively. In free crack banks $f_x(x) = 0$, $f_x(x) = -iq_x(x)$ on the shores of the end areas of the fractures, $f_y(y) = 0$ on the free edges of the cracks that are in the same line with the y-axis, on the edge of the end zones of the cracks $f_y(y) = -iq_y(y)$.

The main ratios of the problem posed should be supported by ratios that combine the slippage of the edge-end regions of the cracks and the forces in the bonds. Without loss of generality, we represent these equations as:

$$u^+(x,\ 0) - u^-(x,\ 0) = C(x,\ q_x(x))q_x(x)$$
$$v^+(0,\ y) - v^-(0,\ y) = C\big(y,\ q_y(y)\big)q_y(y) \tag{4}$$

where, the functions $C(x, q_x(x))$ and $C\big(y, q_y(y)\big)$ are the effective bond conformations, $(u^+ - u^-)$ shifting of the edges of the end zones of the cracks in the same direction as the axis of the abscissa, $(v^+ - v^-)$ shifting of the edges of the end zones of the cracks towards the y-axis.

The boundary conditions in the problem considered to find complex potentials are as follows [10]:

$$\Phi_b(\tau) + \overline{\Phi_b(\tau)} - \left[\bar{\tau}\Phi_b'(\tau) + \Psi_b(\tau)\right]e^{2i\theta} = \Phi_s(\tau) + \overline{\Phi_s(\tau)} - [\bar{\tau}\Phi_s'(\tau) + \Psi_s(\tau)]e^{2i\theta} \tag{5}$$

$$-\kappa_s\overline{\Phi_s(\tau)} + \Phi_s(\tau) - [\bar{\tau}\Phi_s'(\tau) + \Psi_s(\tau)]e^{2i\theta} = \frac{\mu_s}{\mu_b}\left\{-\kappa_b\overline{\Phi_b(\tau)} + \Phi_b(\tau) - \left[\bar{\tau}\Phi_b'(\tau) + \Psi_b(\tau)\right]e^{2i\theta}\right\} \tag{6}$$

$$\Phi_s(t) + \overline{\Phi_s(t)} + t\overline{\Phi_s'(t)} + \overline{\Psi_s(t)} = f_x(t) \quad (7)$$

$$\Phi_s(t_1) + \overline{\Phi_s(t_1)} + t_1\overline{\Phi_s'(t_1)} + \overline{\Psi_s(t_1)} = f_y(t_1) \quad (8)$$

where $\tau = \lambda e^{i\theta} + m\omega_1 + n\omega_2$, $\tau_1 = (\lambda - h)e^{i\theta} + m\omega_1 + n\omega_2$, $(m, n = 0, \pm1, \pm2, ..)$, h is the thickness of the coating, t and t_1 are the additions of the crack edge points in line with the abscissa and ordinate axes, respectively.

Boundary conditions can be simplified [4,5]. It can be shown that the system of reduced equations will be as follows:

$$\left(1 - \frac{\mu_s}{\mu_b}\right)\Phi_b(\tau) + \left(1 + \kappa_b\frac{\mu_s}{\mu_b}\right)\overline{\Phi_b(\tau)} - \left(1 - \frac{\mu_s}{\mu_b}\right)\left[\bar{\tau}\Phi_b'(\tau) + \Psi_b(\tau)\right]e^{2i\theta} = (1 + \kappa_s)\overline{\Phi_s(\tau)}. \quad (9)$$

Therefore, it is necessary to determine three pairs of analytical functions, $\Phi_b(z)$ $\Psi_b(z)$, and $\Phi_s(z)$, $\Psi_s(z)$ from the boundary conditions in Eqs. (5)-(9).

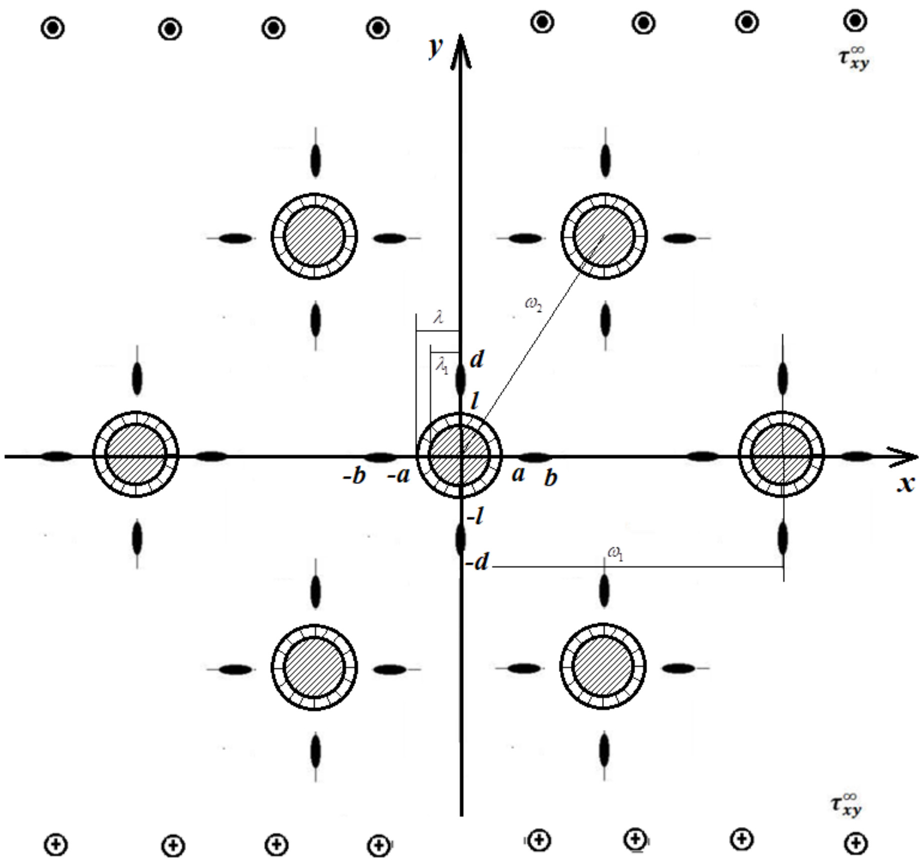

Fig. 1. Calculation diagram of the problem of the interaction of rigid inclusions and cohesive cracks in an isotropic medium under longitudinal shear

3. Solution to the boundary value problem and numerical results

The solution to the boundary value problem is sought as

$$\Phi_s(z) = \Phi_1(z) + \Phi_2(z) + \Phi_3(z), \; \Psi_s(z) = \Psi_1(z) + \Psi_2(z) + \Psi_3(z) \tag{10}$$

$$\Phi_b(z) = i\sum_{k=0}^{\infty} a_{2k}z^{2k}, \quad \Psi_b(z) = i\sum_{k=0}^{\infty} b_{2k}z^{2k} \tag{11}$$

$$\Phi_1(z) = i\tau_{xy}^{\infty} + i\alpha_0 + i\sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}\rho^{(2k)}(z)}{(2k+1)!} \tag{12a}$$

$$\Psi_1(z) = i\tau_{xy}^{\infty} + i\sum_{k=0}^{\infty}\beta_{2k+2}\frac{\lambda^{2k+2}\rho^{(2k)}(z)}{(2k+1)!} - i\sum_{k=0}^{\infty}\alpha_{2k+2}\frac{\lambda^{2k+2}S^{(2k)}(z)}{(2k+1)!} \tag{12b}$$

$$\Phi_2(z) = \frac{1}{2\omega}\int_{L_1} g(t)\, ctg\frac{\pi}{\omega}(t-z)dt \tag{13a}$$

$$\Psi_2(z) = -\frac{\pi z}{2\omega^2}\int_{L_1} g(t)\, sin^{-2}\frac{\pi}{\omega}(t-z)dt \tag{13b}$$

$$\Phi_3(z) = -\frac{i}{2\omega}\int_{L_2} g_1(t_1)\, ctg\frac{\pi}{\omega}(it_1-z)dt_1 \tag{13c}$$

$$\Psi_3(z) = -\frac{i}{2\omega}\int_{L_2}\left\{g_1(t_1)ctg\frac{\pi}{\omega}(it_1-z) + + \left[ctg\frac{\pi}{\omega}(it_1-z) + \frac{\pi}{2\omega}(2t_1+iz)\, sin^2\frac{\pi}{\omega}(it_1-z)\right]g_1(t_1)\right\}dt_1 \tag{13d}$$

Here are $g(t)$ and $g_1(t_1)$ required functions as

$$g(x) = -\frac{2\mu_s i}{1+\kappa_s}\frac{d}{dx}[u^+(x,\;0) - u^-(x,\;0)] \quad \text{length } L_1$$

$$g_1(y) = \frac{2\mu_s}{1+\kappa_s}\frac{d}{dy}[v^+(0,\;y) - v^-(0,\;y)] \quad \text{length } L_2 \tag{14}$$

For plane stress $\kappa_s = 3 - 4v$, for plane strain $\kappa_s = (3-v)/(1+v)$, v Poisson's ratio of the plane material, μ shear modulus, $\rho(z) = \left(\frac{\pi}{\omega}\right)^2 sin^{-2}\left(\frac{\pi}{\omega}z\right) - \frac{1}{3}\left(\frac{\pi}{\omega}\right)^2$, $S(z)$ special meromorphic function [6], $L_1 = \{[-b,-a] + [a,b]\}$ and $L_2 = \{[-d,-l] + [l,d]\}$ are taken along the lines of integrals.

From the anti-symmetry conditions with respect to the coordinate axes, we find:

$$\operatorname{Im}\alpha_{2k} = 0 \qquad \operatorname{Im}\beta_{2k} = 0 \qquad k = 1,\;2,\;\ldots \tag{15}$$

The following follows from the fact that the main vector of all forces acting on the spring connecting two compatible points in region D occupied by the binding material is constant:

$$\alpha_0 = \frac{\pi^2}{24}\beta_2\lambda^2 \tag{16}$$

Additional conditions must be added to the basic integral representations that arise from the physical meaning of the problem such that

$$\int_{-b}^{-a} g(t)dt = 0 \quad \int_{a}^{b} g(t)dt = 0 \quad \int_{-d}^{-l} g_1(t_1)dt_1 = 0 \quad \int_{bl}^{d} g_1(t_1)dt_1 = 0 \tag{17}$$

The unknown $g(t)$ and $g_1(t_1)$ functions and the coefficients a_{2k}, b_{2k}, g_{2k}, h_{2k}, α_{2k}, β_{2k} must be determined from the boundary conditions.

Applying the power series method, we obtain an infinite set of linear algebraic equations.

$$\frac{\mu_s(\kappa_b+1)}{\mu_b} ig_0 = -(\kappa_s+1)\left[i\sum_{j=0}^{\infty}\alpha_{2j+2}\lambda^{2j+2}r_{0,j} - i\tau_{xy}^{\infty} + A_0\right]$$

$$-\left(1-\frac{\mu_s}{\mu_b}\right)i(2k-1)g_{2k}\lambda^{2k} - \left(1+\kappa_t\frac{\mu_s}{\mu_b}\right)ig_{-2k}\lambda^{-2k} - \left(1-\frac{\mu_s}{\mu_b}\right)ih_{-2k-2}\lambda^{2k-2}$$
$$= (\kappa_s+1)\left[i\sum_{j=0}^{\infty}\alpha_{2j+2}\lambda^{2j+2k+2}r_{j,k} + A_{2k}\right]$$

$$\left(1-\frac{\mu_s}{\mu_b}\right)(2k+1)ig_{-2k}\lambda^{-2k} - \left(1+\kappa_b\frac{\mu_s}{\mu_b}\right)ig_{2k}\lambda^{2k} - \left(1-\frac{\mu_s}{\mu_b}\right)ih_{-2k-2}\lambda^{-2k-2} = -(\kappa_s+1)(i\alpha_{-2k} + A_{-2k})$$

$$i\left(g_2\lambda^2 - g_{-2}\lambda^{-2}\right) - ih_0 = -i\sum_{j=0}^{\infty}\alpha_{2j+2}\lambda^{2j+4}r_{1,j} - i\sum_{j=0}^{\infty}\beta_{2j+2}\lambda^{2j+2}r_{0,j} + + i\sum_{j=0}^{\infty}(2j+2)\alpha_{2j+2}\lambda^{2j+2}s_{0,j} - i\tau_{xy}^{\infty} + A_0 \tag{18}$$

$$-i(2k-1)g_{2k}\lambda^{2k} - ig_{-2k}\lambda^{-2k} - ih_{2k-2}\lambda^{2k}$$
$$= -i\sum_{j=0}^{\infty}\beta_{2j+2}\lambda^{2j+2}r_{k-2,j} + + i\sum_{j=0}^{\infty}\alpha_{2j+2}\lambda^{2j+2}r_{j,k}2k\lambda^{2k}$$
$$+ i\sum_{j=0}^{\infty}(2j+2)\alpha_{2j+2}\lambda^{2j+2}s_{k-1,j} + A_{2k}$$

$$i(2k+1)g_{-2k}\lambda^{-2k} - ig_{2k}\lambda^{2k} - ih_{2k-2}\lambda^{-2k-2} = i2k\lambda^{2k} - i\beta_{2j+2} + A_{-2k} \rightleftarrows$$

where $r_{j,k} = \frac{(2j+2k+1)!}{(2j)!(2k+1)!}\frac{g_{j+k+1}}{2^{2j+2k+2}}$, $r_{0,0} = 0$, $s_{j,k} = \frac{(2j+2k+1)!}{(2j)!(2k+1)!}\frac{\rho_{j+k+1}}{2^{2j+2k+2}}$ and $s_{0,0} = 0$.

By requiring the functions in Eq. (8) to satisfy the boundary condition in Eq. (6) at the edges of cracks with end zones $g(x)$ and $g_1(y)$, we obtain two systems of singular integral equations for the functions:

$$\frac{1}{\omega}\int_{L_1} g(t)ctg\frac{\pi}{\omega}(t-x)dt + H(x) = f_x(x)$$

$$-\frac{\pi}{\omega^2}\int_{L_2} g_1(t)\left[(t-y)sh^{-2}\frac{\pi}{\omega}(t-y)dt\right] + N(y) = f_y(y) \tag{19}$$

$$H(x) = x\overline{\Phi_s'(x)} + \overline{\Psi_s(x)},\ \Phi_s(x) = \Phi_1(x) + \Phi_3(x),\ \Psi_s(x) = \Psi_1(x) + \Psi_3(x)$$

$$N(y) = \Phi_0(iy) + \overline{\Phi_0(iy)} + iy\overline{\Phi_0'(iy)} + \overline{\Psi_0(iy)},\ \Phi_0(z) = \Phi_1(z) + \Phi_2(z)$$

$$\Psi_0(z) = \Psi_1(z) + \Psi_2(z)$$

where $ctg\frac{\pi}{\omega}z$, $sh^{-2}\frac{\pi}{\omega}z$ functions in the main period band, the change of variables, and also the quadrature formulas [7,8], we reduce the integral equations to two finite algebra systems with additional conditions in (13). According to approximations and $R_v^0 = g_1^0(\eta_k)$ values of unknown functions at points with Chebyshev nodes $p_k^0 = g^0(\eta_k)$, we have

$$\begin{cases} \sum_{k=1}^{M} A_{m,k} p_k^0 + \frac{1}{2} H_*(\eta_m) = q_x(\eta_m) \quad (m = 1,\ 2,\ \ldots,\ M-1), \\ \sum_{k=1}^{M} \frac{p_k^0}{\sqrt{\frac{1}{2}(1-\lambda_1^2)(\tau_k+1)+\lambda_1^2}} = 0, \end{cases} \tag{20}$$

$$\begin{cases} \sum_{v=1}^{M} A_{m,v} R_v^0 + \frac{1}{2} N_*(\eta_m) = q_y(\eta_m) \quad (m = 1,\ 2,\ \ldots,\ M-1), \\ \sum_{v=1}^{M} \frac{R_v^0}{\sqrt{\frac{1}{2}(1-\lambda_2^2)(\tau_v+1)+\lambda_2^2}} = 0. \end{cases} \tag{21}$$

where $q_x(\eta_m)$ contains the unknown values of shear stresses at the nodes of the end regions, respectively. $q_y(\eta_m)$ contains the unknown stresses in the bonds are determined from additional conditions given by Eq. (3). Using the created solution, we represent Eq. (3) as follows:

$$\begin{aligned} \frac{d}{dx}\left[C(x,\ q_x(x))q_x(x)\right] &= -\frac{(1+\kappa_s)}{2\mu_s i} g(x) \\ \frac{d}{dy}\left[C\big(y,\ q_y(y)\big)q_y(y)\right] &= \frac{(1+\kappa_s)}{2\mu_s} g_1(y) \end{aligned} \tag{22}$$

for L_1 and L_2 respectively. Requiring the conditions in (18) to be fulfilled at the nodes of the tip regions of $q_x(\eta_m)$ and cracks, $q_y(\eta_m)$ $(m = 1,\ 2,\ \ldots,\ M_1)$, we obtain two more systems of equations each to determine the values of M_1. In this case, the finite difference method is used.

The combined solving system of Eqs. (14), (16), (17), (18), p_k^0 $(k = 1,\ 2,\ \ldots,\ M)$, R_v^0 $(v = 1,\ 2,\ \ldots,\ M)$, α_{2k}, β_{2k}, A_{2k}, b_{2k}, h_{2k}, a_{2k} and g_{2k} will be closed to $q_x(\eta_m)$ and $q_y(\eta_m)$ according to the unknowns.

p_k^0 and R_v^0 functions are determined, and the stress intensity coefficients around the crack tips are found using the following relations:

$$\begin{aligned} K_{II}^a &= -i\sqrt{\frac{\pi\ell(1-\lambda_*^2)}{\lambda_*}}\frac{1}{2M}\sum_{k=1}^{M}(-1)^{k+M}p_k^0\, tg\frac{\theta_k}{2}, K_{II}^\ell = -i\sqrt{\pi\ell(1-\lambda_*^2)}\frac{1}{2M}\sum_{k=1}^{M}(-1)^{k}p_k^0 c\, tg\frac{\theta_k}{2} \\ K_{II}^b &= -i\sqrt{\frac{\pi r(1-\lambda_2^2)}{\lambda_2}}\frac{1}{2M}\sum_{v=1}^{M}(-1)^{v+M}R_v^0\, tg\frac{\theta_v}{2}, K_{II}^r = -i\sqrt{\pi r(1-\lambda_2^2)}\frac{1}{2M}\sum_{v=1}^{M}(-1)^{v}R_v^0 c\, tg\frac{\theta_v}{2} \end{aligned} \tag{23}$$

We note the limiting cases. Inclusion, coating, and binding materials are the same; $\mu_b = \mu_s$, $\kappa_b = \kappa_s$. From the given solution it follows:

$$\Phi_1(z) = i\tau_{xy}^{\infty},\ \Psi_1(z) = i\tau_{xy}^{\infty},\ \Phi_b(z) = 0,\ \Psi_b(z) = 0 \tag{24}$$

Then, we obtain a solution for rows of cracks without holes.

In $\omega \to \infty$, the general notation of Eqs. (10)–(13) presents a solution to the problem of a single insert uniformly coated with a homogeneous film with cracks along the coordinate axes for the binder. To obtain a solution for a single hole with a crack, it is also necessary to take $\frac{\mu_b}{\mu_s} = 0$.

To analyze the limit stability of end-zone cracks, two conditions (two-parameter criteria) for failure are required. The first criterion is the progress of the crack tip, and the second is the breaking state of the bonds at the edge of the tip region.

As the initial failure condition, we use the Irwin force criterion for failure. The limit equilibrium state of the crack tip corresponds to the fulfillment of the condition

$$K_{II} = K_{IIc} \tag{25}$$

where K_{IIc} is the constant of the material determined empirically from [2,9].

As the second failure condition, we use the criterion of critical sliding of crack edges and consider the breaking of bonds at the edge of the end zone ($x_* = \ell - d$ or $y_* = r - d_1$) to occur when the condition is met as

$$V(x_*) = \sqrt{(u^+ - u^-)^2 + (v^+ - v^-)^2} = \delta_{cr} \tag{26}$$

where is δ_{cr} the maximum bond length.

The solution of the system of algebraic equations (16), (17), (18), (20), and (21) for a given crack length and bond properties is the limit slip of the end region in the case of critical external load and the limit crack balance τ_{xy}^{∞}.

For cracks of certain sizes and end zones, K_{II} and K_{IIc} using the limit values, and it is possible to determine the equilibrium and growth modes of cracks under monotonic loading when the conditions are satisfying

$$V(x_*) \quad \text{and} \quad V(y_*) < \delta_{cr} \tag{27}$$

The crack tip then progresses with a simultaneous increase in the length of the tip region without severing the bonds. This stage of crack development can be considered as a process of adaptation to a certain level of external load.

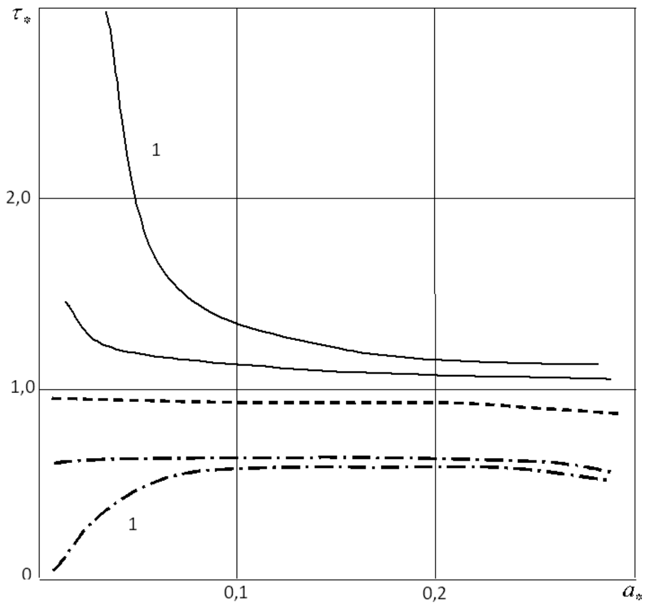

Fig. 2. Distance dependencies of $a_* = a - \lambda$ critical load for both ends of the crack in $\tau_* = \tau_{xy}^{\infty}\sqrt{\omega}/K_{IIc}$ ($\lambda = 0.3$)

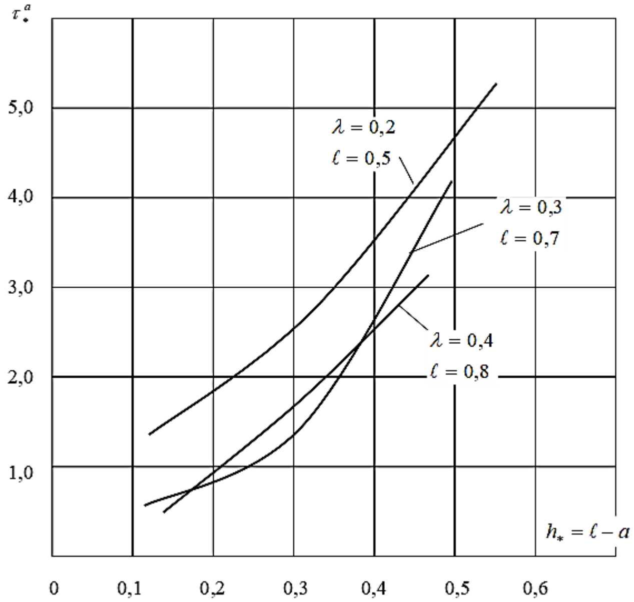

Fig. 3. Dependence $\tau_*^a = \tau_{xy}^{\infty}\sqrt{\omega}/K_{IIc}$ of ultimate load $h_* = \ell - a$ on crack length

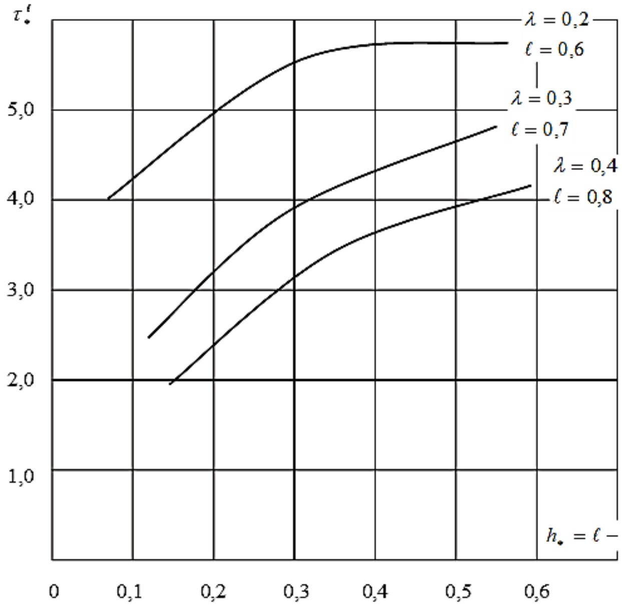

Fig. 4. $\tau_*^\ell = \tau_{xy}^{\infty}\sqrt{\omega}/K_{IIc}$ dependence of ultimate load on A2 crack length

Growth of the crack tip with a simultaneous rupture of bonds at the edge of the tip zone occurs when the following conditions are satisfying:

$$K_{II} \geq K_{IIc} \quad V(x_* \text{ or } y_*) \geq \delta_{cr} \tag{28}$$

The bonds break before the crack tip advances, and the size of the tip zone decreases, tending to a critical value for a certain load level. When conditions are satisfying

$$K_{II} < K_{IIc} \quad V(x_* \text{ or } y_*) < \delta_{cr} \tag{29}$$

The positions of the crack tip and end zone will not change.

Fig. 2 shows the dependence of the $\tau_* = \tau_{xy}^{\infty}\sqrt{\omega}/K_{IIc}$ critical load on its distance $a_* = a - \lambda$ for both ends of the crack along the abscissa axis (curve 1 corresponds to the left end). For comparison, the dashed line τ_* shows the dependence of inclusions and in the absence of coatings (the material of the inclusion, coating, and binder is the same) with the same fracture geometry calculated by the described method. In the same place, the dashed line indicates the dependency in the case of absolutely flexible inclusion (the holes are not filled with anything). For any elastic inclusion, the stress state pattern will occupy an intermediate position between these two limiting states.

Figs. 3 and 4 show graphs of the dependence of the ultimate load on the length of the crack for $\mu_b/\mu_s = 25$. As can be seen, λ at certain values of the hole radius, stable development of a crack system (their mutual strengthening) is possible. Parametric analysis of the problem showed that the stress concentration near inclusions in the binder has a significant effect on the development of very small cracks. With increasing length of cracks with end zones, this effect disappears, and $\ell - \lambda > \lambda$ and $r - \lambda > \lambda$ can already be neglected in and, but in this case, the interaction of cracks begins to be affected. Depending on the geometric and physical parameters of the problem, a stable crack development with end zones is observed. The presence of a flexible inclusion increases the stress intensity factor, while rigid inclusions reduce it compared to the binding material.

4. Conclusion

The inclusion effect is especially effective at closely spaced crack tips. Thus, analysis τ_{xy}^{∞} shows that the magnitude of the external load and K_{IIc}, δ_{cr} its critical parameters determine the nature of the destruction:

1. growth of the crack tip with advancement of the tip zone,
2. reducing the size of the tip zone without expanding the crack tip,
3. growth of the crack tip with simultaneous rupture of bonds at the edge of the tip region.

The model of a crack with end zones makes it possible to study the patterns of force distribution in ties under various laws of deformation, analyze the boundary stability of cracks taking into account deformation and force failure criteria, and also evaluate the critical external load. and crack resistance of the composite body (composite). It is possible to select a system of sealants (inclusions) in such a way that the stress field they create slows down the development of cracks in the binder.

Conflict of interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

This research received no external funding.

Data availability statement

Data generated during the current study are available from the corresponding author upon reasonable request.

References

[1] Barenblatt GI, Cherepanov GP (1975) System of arbitrarily oriented longitudinal shear cracks in an elastic solid: PMM vol. 39, pp. 717–723. Journal of Applied Mathematics and Mechanics 39(4):685-692.

[2] Panasyuk VV (1991) Mechanics of quasi-brittle fracture of materials. Naukova Dumka, Kiev (in Russian).

[3] Pestrikov VM, Morozov EM (2002) Fracture mechanics of solids. Ed. "The Professional" SPb (in Russian).

[4] Wang GA (1971) Theory of reinforced materials. Naukova Dumka, Kiev (in Russian).

[5] Vanin GA (1985) Micromechanics of composite materials. Naukova Dumka, Kiev (in Russian).

[6] Mirsalimov VM (1984) Fracture of elastic and elastic-plastic bodies with cracks. Science, Baku.

[7] Kalandiya AI (1959) On the approximate solution of a class of singular integral equations. Dokl. Akad. Nauk SSSR 125:715–718 (in Russian).

[8] Panasyuk VV, Savruk MP, Datsyshin AP (1976) Stress distribution near cracks in plates and shells. Naukova Dumka, Kiev (in Russian).

[9] Panasyuk VV, Andreikiv AE, Parton VZ (1988) Fracture mechanics and strength of materials. Naukova Dumka, Kiev (in Russian).

[10] Muskhelishvili NI (1966) Some Basic Problems of the Mathematical Theory of Elasticity. Nauka, Moscow (in Russian).

[11] Mehtiyev RK (2018) On interaction of hard inclusions and cohesion cracks in the isotropic environment under the longitudinal shift. In: Proceedings of the 6th International Conference and Optimization with Industrial Applications.

[12] Mehtiyev RK (2020) Interaction of a two-periodic system of foreign elastic inclusions, the surface of which is uniformly covered by a homogeneous cylindrical film, and two systems of rectilinear cracks. In: Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications.

[13] Mehtiyev RK (2020) Longitudinal shift of bodies with complex structure, relaxed rectified through crack. In: Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications.

[14] Babanli M, Turen Y, Gurbanov N, Mehtiyev R, Askin M, Ismayilov M (2022) Theory and experiment in predicting the strength of hybrid fiber metal laminates. Journal of Applied and Computational Mechanics 9(4):987-999.

[15] Yaylaci M, Abanoz M, Uzun Yaylaci E, Olmez H, Sekban DM, Birinci A (2022) The contact problem of the functionally graded layer resting on rigid foundation pressed via rigid punch. Steel and Composite Structures 43(5):661-672.

[16] Yaylacı M, Öner E, Adıyaman G, Öztürk Ş, Uzun Yaylacı E, Birinci A (2023) Analyzing of continuous and discontinuous contact problems of a functionally graded layer: theory of elasticity and finite element method. Mechanics Based Design of Structures and Machines 6(7):540-551.